



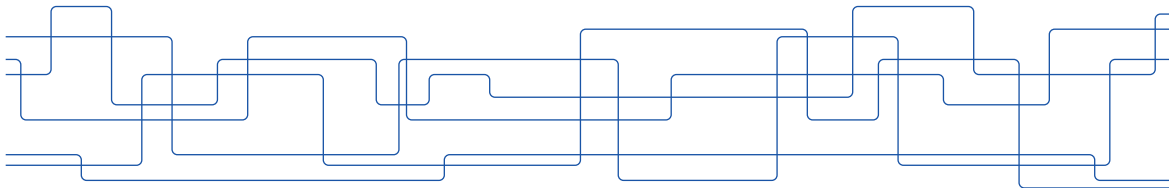
## Mining temporal networks

Lecture in course “Social networks and online markets”

Sapienza, Wednesday, April 10, 2024

*Aristides Gionis, KTH Royal Institute of Technology, Sweden*

`argioni@kth.se`



# agenda

part I : introduction and motivation

part II : models of temporal networks

part III : algorithmic frameworks

part IV : data mining problems

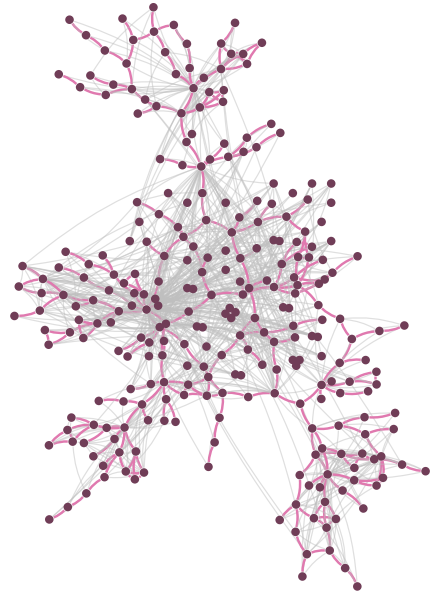
part V : conclusions and future challenges

part I

introduction and motivation

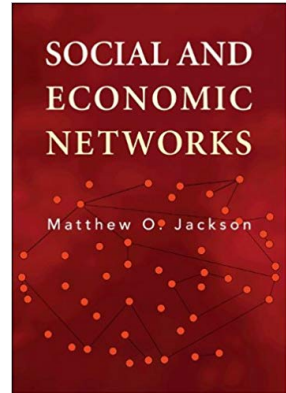
# interconnected world

- networks model **objects** and their **relations**
- many different **network types**
  - social
  - informational
  - technological
  - biological
  - ...



# impact of network science

- online communication networks and social media
- implications in
  - knowledge creation
  - information sharing
  - education
  - democracy
  - society as a whole



## research questions in network science

- structure discovery
  - communities, summarization, events, role mining
- study complex dynamic phenomena
  - evolution, information diffusion, opinion formation, structural prediction
- develop novel applications
- design efficient algorithms

## traditional view

- networks represented as pure graph-theory objects  
no additional vertex / edge information
- emphasis on **static networks**
- **dynamic** settings model **structural changes**  
vertex / edge additions / deletions

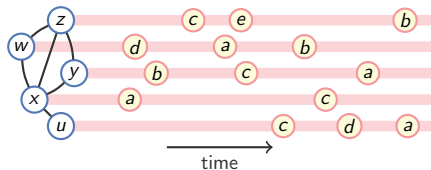
## temporal networks

- ability to collect and store large volumes of network data
- available data have **fine granularity**
- lots of **additional information** associated to vertices/edges
- network topology is **relatively stable**, while lots of **activity** and **interaction** is taking place
- giving rise to **new concepts**, **new problems**, and **new computational challenges**

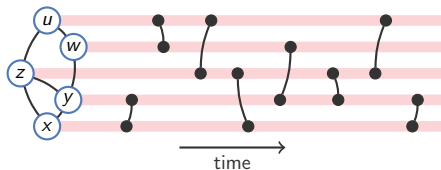


## modeling activity in networks

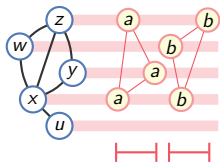
1. network nodes **perform actions** (e.g., posting messages)



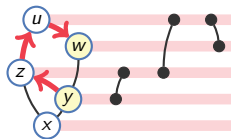
2. network nodes **interact** with each other  
(e.g., a "like", a repost, or sending a message to each other)



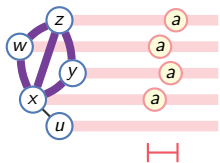
many novel and interesting concepts



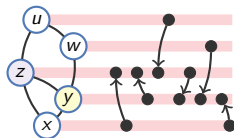
new pattern types



temporal information paths



new types of events



network evolution

## temporal networks — objectives

- identify new concepts and new problems
- develop algorithmic solutions
- demonstrate relevance to real-world applications

# terminology

- we use term “temporal networks”, but terminology is not standardized
- term “X Y” can be encountered in the literature, where

X:

temporal  
dynamic  
(time-)evolving  
time-varying  
time-dependent  
evolutionary

Y:

networks  
graphs

# examples of temporal networks

[Holme, 2015]

- **human communication networks**
  - phone, email, text messages, etc.
- **human proximity networks**
  - recorded by various sensors and devices, e.g., bluetooth, wifi, etc.
  - patient-referral networks, i.e., how patients are transferred between wards of a hospital system
  - sexual contact networks
- **animal proximity networks**
  - obtained via RFID devices
  - livestock or wildlife

## examples of temporal networks — cont'd

[Holme, 2015]

- **bibliographic networks**
  - collaboration and citation networks
- **economic networks**
  - credit card transactions
  - trade networks of countries
  - bitcoin transactions
- **travel and transportation networks**
  - airline connections, bus transport, bike-sharing systems

## examples of temporal networks — cont'd

[Holme, 2015]

- **brain networks**
  - temporal correlations of the oxygen levels of brain regions as measured by fMRI scanning
- **biological networks**
  - genes involved in different interactions that change over time
  - current challenges, as one cannot measure precisely when two proteins interact with each other, but technology is improving

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part IV : data mining problems

part V : conclusions and future challenges



part II

models of temporal networks

# representation of temporal networks

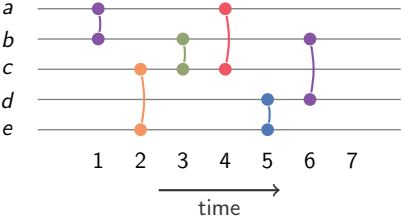
## 1. sequence of interactions

- a temporal network is represented as  $G = (V, E)$ 
  - with set of nodes  $V$ , and set of edges  $E = \{(u, v, t)\}$ , with  $u, v \in V$  and  $t \in \mathbb{R}$
  - if interactions have duration, then  $E = \{(u, v, t, \lambda)\}$
- this is a lossless representation — no information is lost
- also known as sequence of contacts, or sequence of (temporal) edges

# representation of temporal networks

## 1. sequence of interactions

- visual representation of a temporal network as a sequence of interactions



# representation of temporal networks

## 2. sequence of static graphs

- sequence  $G_1, \dots, G_T$

where  $G_t = (V_t, E_t)$ , with  $t = 1, \dots, T$

typically assume that nodes are fixed, i.e.,  $V_t = V$

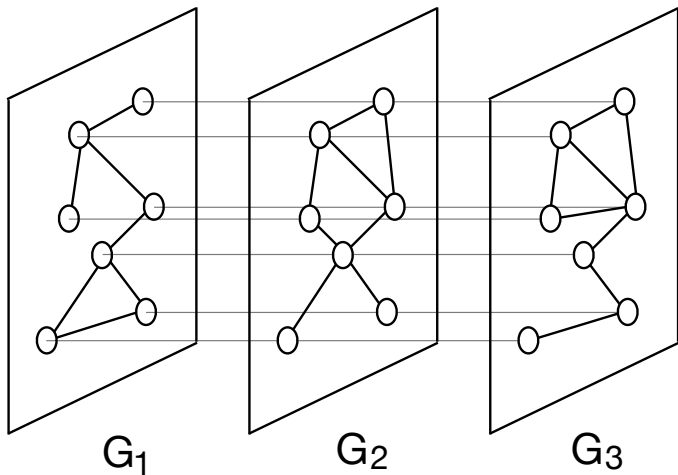
$E_t$  are the edges that occur in time interval  $t$

- **advantages:** static graph analysis methods can be applied
- **disadvantages:** the representation assumes quantization into time intervals
  - thus, representation depends on quantization parameters, e.g., seconds, minutes, hours, days, etc.
  - coarse resolution may lead to information loss
  - fine resolution may lead to sparse (or even empty) static graphs

## representation of temporal networks

### 2. sequence of static graphs

- visual representation of a temporal network as a sequence of static graphs



# representation of temporal networks

## 3. time series of contacts

- a time-series for each pair of nodes in the network
- equivalent representation with sequence of interactions

## 4. tensor representation

- tensor representing  $\text{node} \times \text{node} \times \text{time}$  information
- can apply powerful tensor-algebra techniques
- a complication is that time is directed, while tensor algebra assumes that indices can be relabeled (breaking the time ordering)

## representation of temporal networks

[Casteigts et al., 2012]

5. **time-varying graphs** defined as  $G = (V, E, T, p, \lambda)$ , where
  - $V$ : set of nodes
  - $E \subseteq V \times V$ : set of edges
  - $T$ : a time domain
  - $p : E \times T \rightarrow \{0, 1\}$ : a presence function
  - $\lambda : E \times T \rightarrow \mathbb{R}$ : a latency function
- general definition that can be used to model graph datasets in different applications
  - transportation networks, communication networks, social networks

## temporal networks vs. dynamic graphs

- **dynamic graphs** is a standard model typically studied in **theoretical computer science**
  - e.g., [Henzinger et al., 1999, Thorup, 2000]
- dynamic graphs are represented as a **sequence** of **edge additions** and/or **edge deletions**
- $G_0$  is the initial graph, and  $G_i$  is the graph resulting after the  $i$ -th edge addition/deletion operation
- **objective:** **efficient maintenance of graph properties**
  - e.g., connectivity, shortest paths, spanners, etc.



## temporal networks vs. dynamic graphs

- in studies of dynamic graphs, the properties of interest refer to **individual graph snapshots**  $G_i$ , not considering the whole **graph evolution**
- emphasis on **computational efficiency**
  - computation time **per operation**
  - e.g., cost of maintaining a minimum spanning tree per edge additions/deletions
  - or, cost of maintaining a data structure that allows to answer short-path queries
- **dynamic graph** model captures **topological changes**, not interactions
  - e.g., dynamic graphs can be used to model friendship additions/deletions in a social network, but not discussions or other interactions

## temporal networks vs. dynamic graphs

- **dynamic graphs** resemble **sequence of interactions** model
- main difference lies on which **graph properties** we study
- for dynamic graphs we typically consider **properties on graph snapshots**
  - i.e., minimum spanning tree on the current snapshot
- for temporal graphs we typically consider **properties that span a time interval**
  - i.e., a temporal pattern
- **note**: here we focus more on temporal networks, not so much on dynamic graphs

## graph streams

- setting inspired by **data streams** [Muthukrishnan et al., 2005]
- recall the **data-stream model**:
  - data are presented as a **sequence of data items** (potentially infinite)
  - assume a **small number of passes** typically constant or just one pass
  - assume **small memory** compared to data size, e.g., poly-logarithmic
  - assume **fast computation** per data item processed, e.g., constant or poly-logarithmic

## graph streams

- a **graph stream** is a **graph dataset** in the **data-stream model**
  - it can be **either** a sequence of interactions (temporal network)  
**or** sequence of edge additions/deletions (dynamic graph)
- thus, a graph stream is **not** a **representation model**, instead it refers to the underlying **computational model**
- thus, we can study questions of mining temporal networks in the graph-stream model

## dynamic graph algorithms on streaming model

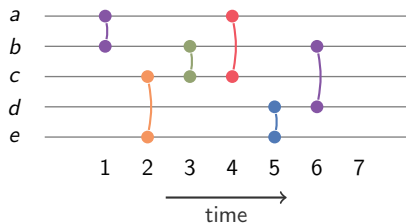
- well-studied model
- extensive survey by [McGregor, 2014]
- different settings considered
  - node/edge additions (**incremental**)
  - node/edge additions/deletions (**fully-dynamic**)
  - updating weights/labels is a special case of the fully-dynamic model
  - **sliding-window setting**: consider only edges from latest interval of fixed length
  - algorithms can be **deterministic** or **randomized**

time-respecting paths

## time-respecting paths

- a **fundamental concept** in analysis of temporal networks
  - used in studies of **information propagation**, or **epidemics spreading**
- a **time-respecting path** is a **sequence of temporal edges**, such that
  - consecutive edges **share a common node**, and
  - time stamps of temporal edges are **non-decreasing**
- intuitively, a piece of information (or disease) can propagate in the network **only** over **time-respecting paths**

## time-respecting paths — example



$(c, e, 2), (e, d, 5), (d, b, 6)$  is a time-respecting path from *e* to *b*

$(c, b, 3), (b, a, 1)$  is not a time-respecting path



## static expansion of a temporal network

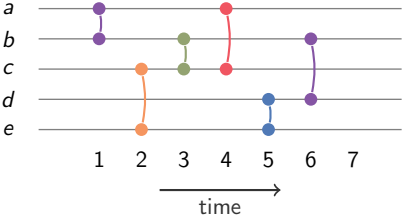
- a transformation of a temporal network to a directed (static) network so that time-respecting paths in the temporal network correspond to directed (static) paths in the directed (static) network
- how to create such a transformation?

## static expansion of a temporal network

- a transformation of a temporal network to a directed (static) network so that time-respecting paths in the temporal network correspond to directed (static) paths in the directed (static) network
- how to create such a transformation?
  1. create a copy of each node for each time instance
  2. create a directed edge from the  $(t - 1)$ -th copy of  $u$  to the  $t$ -th copy of  $u$ , for all nodes  $u$ , and all time instances  $t$
  3. create directed edges for the temporal edges

# static expansion of a temporal network

example



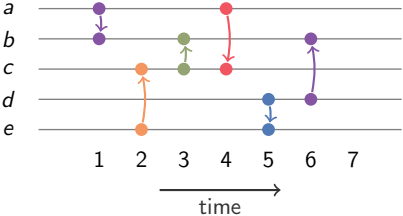
(a) representation of a temporal network



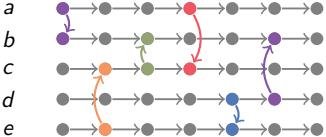
(b) static expansion of temporal network;

# static expansion of a temporal network

example



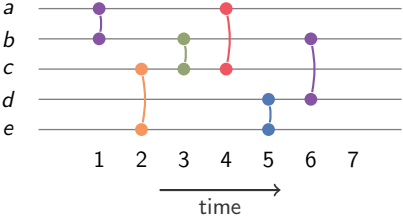
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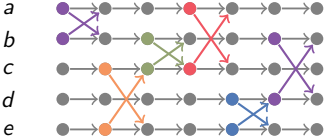
(b) static expansion of temporal network;  
directed edges

# static expansion of a temporal network

example



(a) representation of a temporal network



(b) static expansion of temporal network;  
delays

## reachability, connectivity, and connected components

- defined as in static graphs, but using **time-respecting paths**
- **reachability**:
  - used to study **infection spreading** and **information cascades**
- **connectivity**: as in directed (static) graphs is not symmetric
  - distinguish **strong** and **weak** connectivity
  - in addition, we can define **transitive connectivity**:  
a subgraph is transitively connected if time-respecting paths from  $u$  to  $v$   
and  $v$  to  $w$  imply a time-respecting path from  $u$  to  $w$

## minimum temporal paths

different notions of **minimum temporal paths** rely on **time-respecting paths**

- **earliest-arrival path**: a path from  $x$  to  $y$  with earliest arrival time
- **latest-departure path**: a path from  $x$  to  $y$  with latest departure time
- **fastest path**: path from  $x$  to  $y$  with minimum elapsed time
- **shortest path**: fastest path from  $x$  to  $y$  in terms of overall traversal time required on edges

[Wu et al., 2014]

## diameter, network efficiency

- **diameter**: shortest latency of time-respecting paths over connected pairs [Chaintreau et al., 2007]
  - restricted on connected pairs, as real data have many disconnected pairs
- **network efficiency**: the harmonic mean of latency over all pairs [Tang et al., 2009]
  - **discussion**: what is the motivation for **harmonic mean**?



## diameter, network efficiency

- **diameter**: shortest latency of time-respecting paths over connected pairs [Chaintreau et al., 2007]
  - restricted on connected pairs, as real data have many disconnected pairs
- **network efficiency**: the harmonic mean of latency over all pairs [Tang et al., 2009]
  - **discussion**: what is the motivation for **harmonic mean**?
  - it combines average latency and reachability ratio

## centrality measures

- many centrality measures on static graphs use distances
- **closeness centrality**:  $C_c(u) = \frac{n-1}{\sum_{v \neq u} d(u,v)}$
- **betweenness centrality**:  $C_b(u) = \frac{\sum_{v \neq u \neq w} p_u(v,w)}{\sum_{v \neq u \neq w} p(v,w)}$
- for temporal networks we replace distance with shortest latency time-respecting path
- analogues of **Katz centrality** and **PageRank** have also been defined
- **discussion**: how do these centrality measures on temporal networks compare with their static analogues?
  - main difference is that the temporal information becomes relevant  
e.g., betweenness centrality refers to **a given node at a given time**

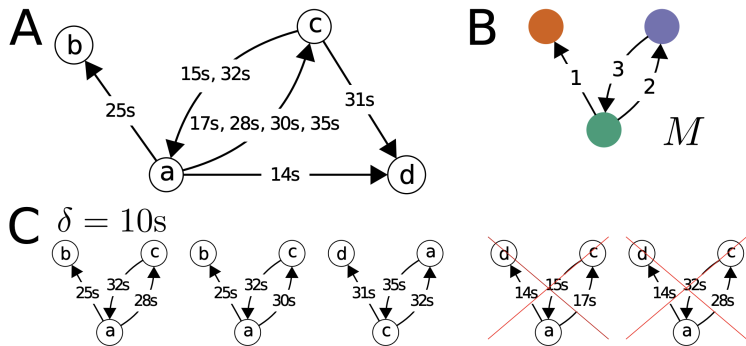
# temporal motifs

- temporal motif counting

[Paranjape et al., 2017, Kovanen et al., 2013]:

- temporal motif is a **small subgraph** with **temporally ordered** edges  
(and/or interval or delay constraints)

## temporal motifs



$\delta$ -temporal motif: a sequence of directed temporally ordered edges which appear within a time window  $\delta$

[Paranjape et al., 2017]

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part III

algorithmic frameworks for temporal network analysis

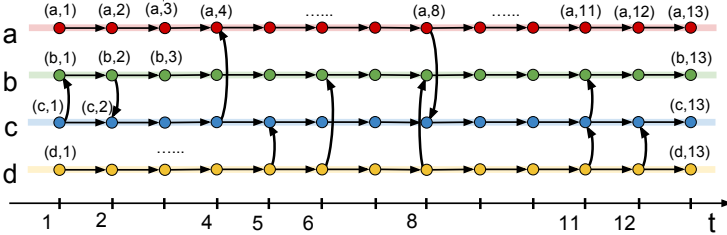
# frameworks

## adopted traditional frameworks

- static expansion graphs
- dynamic graphs
- time-series

# static expansion graphs

- static graph of time-stamped nodes and time-forwarding edges  $G_e = (V_e, E_e)$
- $V_e = \{(v, t) \mid v \in V, t \in T\}$ , where  $T$  is the set of all possible timestamps
- edges  $E_e$  : interactions between the temporal nodes  $V_t$





## static expansion graphs

- static expansion graph is a **directed graph**
- **standard graph algorithms** (BFS, DFS, Dijkstra, Bellman-Ford) can be adopted for finding:
  - fastest temporal paths, shortest temporal paths, and weighted combinations
  - walks (revisiting of vertices and edges is allowed)
  - journeys (revisiting of vertices is allowed, but not edges)
- **upstream**, **downstream** reachability sets

## minimum temporal paths

different notions of **minimum temporal paths** rely on **time-respecting paths**

- **earliest-arrival path**: a path from  $x$  to  $y$  with earliest arrival time
- **latest-departure path**: a path from  $x$  to  $y$  with latest departure time
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[Wu et al., 2014]

## earliest-arrival path

- temporal graph  $G = (V, E)$
- source vertex  $x$ , starting time  $t_s$
- array  $T$  of size  $|V|$  to record arrival times to each node
- $T[x] = t_s$  and  $T[v] = \infty$ , for nodes other than source
- process edges  $(u, v, t, \lambda)$  in temporal order
  - if  $t \geq T[u]$  ( $u$  is already reached from  $x$ )
  - check if the edge creates the earliest-seen-so-far path from  $x$  to  $v$  and update  $T[v]$ :  
 $T[v] = \min(T[v], t + \lambda)$

[Wu et al., 2014]

## latest-departure path

- temporal graph  $G = (V, E)$
- sink vertex  $x$ , ending time  $t_s$
- same process as for earliest-arrival path, but
  - process edges in reverse temporal order
- add new interaction to the path if it does not violate temporal order

[Wu et al., 2014]

## minimum spanning trees

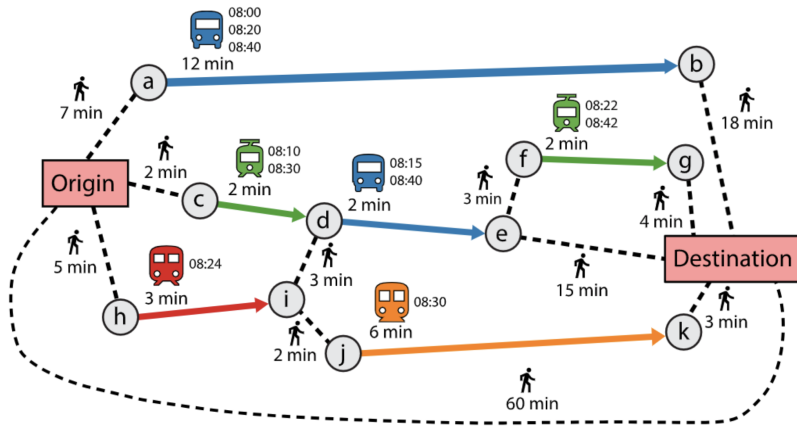
- $mst_a$  : minimum spanning tree with earliest-arrival times
  - each temporal path from the root to the node is the earliest arrival path
  - can be solved in linear time
- $mst_w$  : minimum spanning tree with smallest total weight
  - or with the smallest number of hops: directed Steiner tree
  - NP-hard, can adapt approximation algorithm from directed Steiner tree

[Huang et al., 2015]

## applications of temporal paths

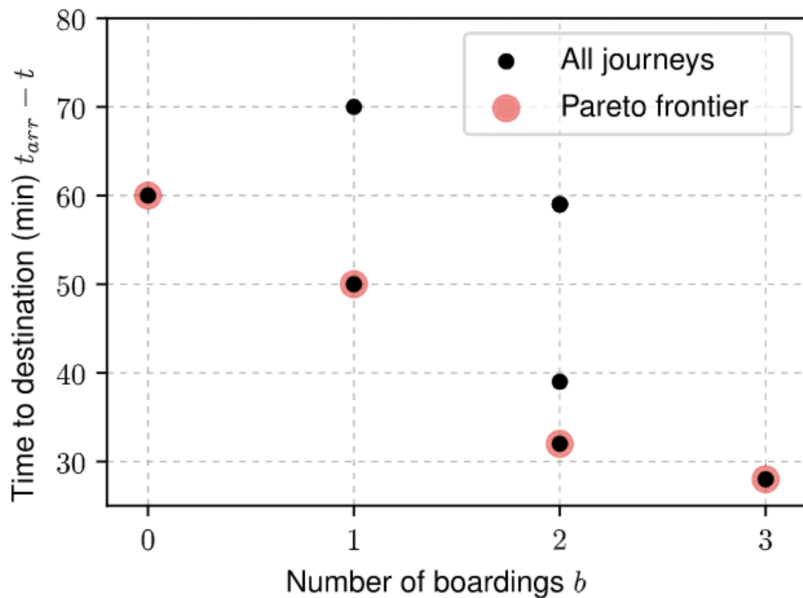
- temporal reachability problems
  - diffusion simulation, centrality measures
- directed spanning trees or Steiner trees
  - reconstruction of diffusion
- **drawback**: large size of expansion graph may lead to high computational complexity and large memory consumption
- **challenge**: **scalable algorithms** and **approximations**

# applications — transportation temporal networks



[Kujala et al., 2018]

## Pareto-optimal journeys





## dynamic graph algorithms on streaming model

- well-studied model
- extensive survey by [McGregor, 2014]
- different settings considered
  - node/edge additions (**incremental**)
  - node/edge additions/deletions (**fully-dynamic**)
  - updating weights/labels is a special case of the fully-dynamic model
  - **sliding-window setting**: consider only edges from latest interval of fixed length
  - algorithms can be **deterministic** or **randomized**

	Insert-Only	Insert-Delete	Sliding Window (width $w$ )
Connectivity	Deterministic [27]	Randomized [5]	Deterministic [22]
Bipartiteness	Deterministic [27]	Randomized [5]	Deterministic [22]
Cut Sparsifier	Deterministic [2, 8]	Randomized [6, 31]	Randomized [22]
Spectral Sparsifier	Deterministic [8, 46]	Randomized $\tilde{O}(n^{5/3})$ space [7]	Randomized $\tilde{O}(n^{5/3})$ space [22]
$(2t - 1)$ -Spanners	$O(n^{1+1/t})$ space [11, 23]	Only multiple pass results known [6]	$O(\sqrt{wn^{(1+1/t)}})$ space [22]
Min. Spanning Tree	Exact [27]	$(1 + \epsilon)$ -approx. [5] Exact in $O(\log n)$ passes [5]	$(1 + \epsilon)$ -approx. [22]
Unweighted Matching	2-approx. [27] 1.58 lower bound [42]	Only multiple pass results known [3, 4]	$(3 + \epsilon)$ -approx. [22]
Weighted Matching	4.911-approx. [25]	Only multiple pass results known [3, 4]	9.027-approx. [22]

**Table 1: Single-Pass, Semi-Streaming Results: Algorithms use  $O(n \text{ polylog } n)$  space unless noted otherwise.**

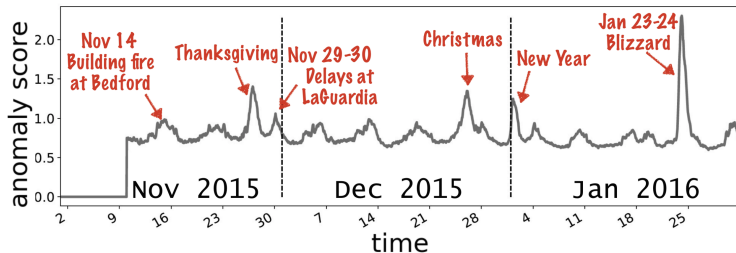
## time-series analysis

- view a temporal network as a (multivariate) time series
  - calculate temporal profile of nodes, edges, or a whole network
  - calculate distance between adjacent snapshots and analyze the resulting time series
- distance: edit distance, node-profile distances, vector-space distance
- applications in change-point detection, anomaly detection, evolutionary pattern mining

## event detection in time series

- given a sequence of graphs  $G_t$
- a function to calculate the vertex affinity matrix  $S$ , where  $s_{ij}$  indicates the influence vertex  $i$  has on vertex  $j$
- a set of time points for detected events is  $\{t \in T \mid d(G_t, G_{t+1}) \geq \delta\}$  where

$$d(G_t, G_{t+1}) = \sqrt{\sum_{i=1}^n \sum_{j=1}^n \left( \sqrt{s_{ij}^{(t)}} - \sqrt{s_{ij}^{(t+1)}} \right)^2}$$



## time-series analysis

- anomaly detection survey [Ranshous et al., 2015]
- approach does not solve all the problems, as it does not capture the network topology
- possible work-around: use more topology embeddings metrics
  - larger neighborhoods, influence measures, eigenvectors, ...

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part IV

data mining problems

## data mining problems

- community detection
- event detection
- finding important nodes
- epidemics analysis and influence spreading



community detection

## community detection in static graphs

- static graphs: extensive survey [Fortunato and Hric, 2016]
- standard community definitions
  - a community is a set of nodes, which are closer to each other than to the rest of the network
  - a community is a dense network subgraph
- general definition [Coscia et al., 2011]
  - a community in a complex network is a set of entities that share some closely correlated sets of actions with the other entities of the community
- typical problem settings
  - a single community vs. network partition
  - overlapping vs. non-overlapping communities

# community detection in static graphs

## partition measures

- **modularity**: the difference between the actual number of edges and the expected
- **cut**: number of edges between a community and the rest of the graph
- **ratio cut**: cut normalized by the number of edges of community nodes
- ...

## single-community measures

- average degree:  $\frac{|E(S)|}{2|S|}$
- density:  $\frac{2|E(S)|}{|S|(|S|-1)}$
- conductance:  $\frac{cut(S, \bar{S})}{\min\{vol(S), vol(\bar{S})\}}$
- ...

# community detection in temporal networks

temporal information gives rise to several issues

- **temporal localization**: concise time interval or intervals, whole time history
- **behaviour**: single-appearance, recurring, persistent, evolutionary patterns, smoothness
- partition of the **topology network** vs. partition of the **time history**
- **online** vs. **offline**
- application-specific settings

## temporal communities: temporal assumptions

often assume a **prior model**, which describes what is the temporal behavior of interesting community structures, e.g.,

- small/large covering intervals of community interactions
- frequent patterns
- persistent patterns

## evolutionary patterns: vocabulary

evolutionary patterns of communities in the network

[Dakiche et al., 2019]

- birth
- death
- growth
- contraction
- merge
- split
- continue
- resurgence

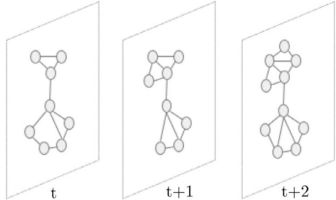
## temporal communities: idea #1

we follow a recent survey on community detection

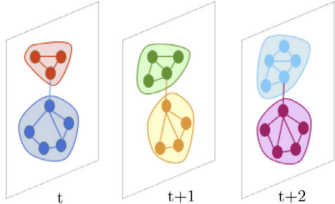
[Dakiche et al., 2019]

- independent community detection and matching
  - first detect communities at each timestamp
  - then match them across different timestamps

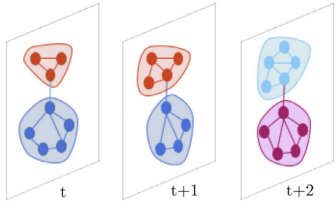
# independent community detection and matching



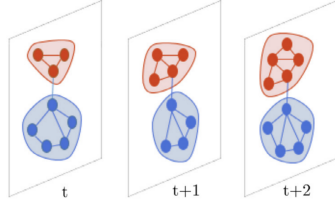
(1) A dynamic network consisting of three snapshots



(2) Community detection in each snapshot



(3) Match communities of  $t$  and  $t+1$

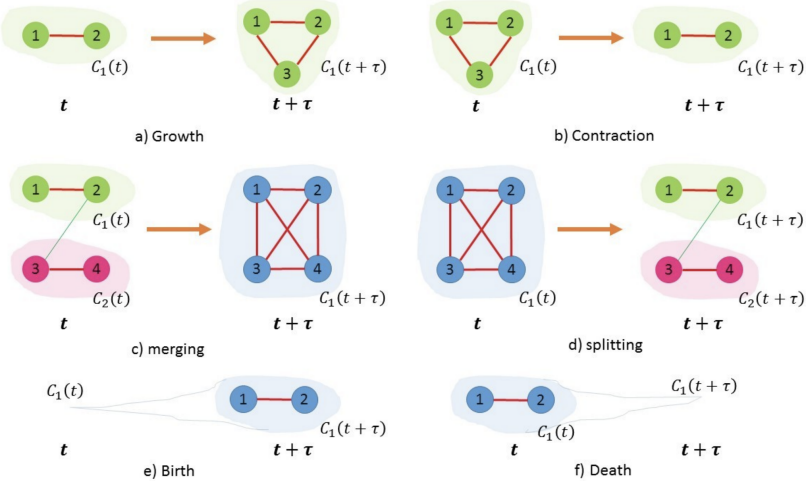


(4) Match communities of  $t+1$  and  $t+2$





# typical evolutionary patterns



[Sun et al., 2015]

# independent community detection and matching

## advantages

- reuse of standard community detection methods
- use existing similarity measures

## disadvantages

- instability of community-detection algorithms

## temporal communities: idea #2

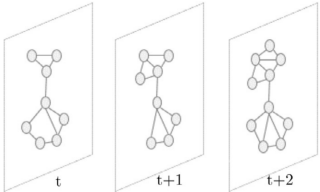
- dependent community detection

[Dakiche et al., 2019]

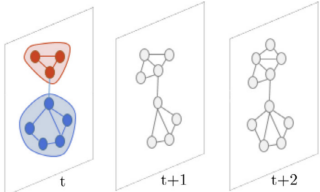
detect communities at time  $t$  based on

- network topology at  $t$ , and
- communities at time  $t - 1$

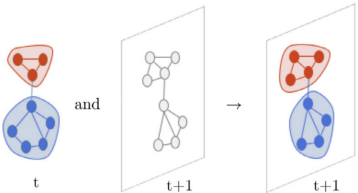
# dependent community detection



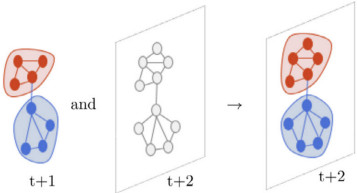
(1) A dynamic network consisting of three snapshots



(2) Community detection in the first snapshot



(3) Community detection at  $t+1$  using graph of  $t+1$  and communities of  $t$



(4) Community detection at  $t+2$  using graph of  $t+2$  and communities of  $t+1$

[Dakiche et al., 2019]

# Louvain algorithm

- a fast greedy approach based on **modularity optimization**
- **reminder**: the **modularity** objective

$$Q = \frac{1}{2m} \sum_{u,v} \left[ A_{uv} - \frac{d_u d_v}{2m} \right] \delta(c_u, c_v)$$

- **two phases repeated iteratively**
  - initially, each node in network is a community
  - then, for each node  $i$ , consider its neighbor  $j$  and compute the gain of modularity of putting  $i$  into the community of  $j$
  - node  $i$  is placed into the community with the largest gain, if the gain is positive

[Blondel et al., 2008]

## Louvain algorithm

- on the **second phase**, each community is considered as a super-node
  - the edges between these super-nodes are contracted and re-weighted by the number of edges between them
- the two phases are repeated until there is **no improvement** in modularity
- the algorithm is **extremely fast**

[Blondel et al., 2008]

## history-dependent approach

### idea

- for two consecutive time steps, there only few edges that affect the community structure
- if the connections of all the nodes in the same community at time step  $t - 1$  keep unchanged at time step  $t$ , they are still in the same community at time step  $t$
- thus, no need to break that super-node

[He and Chen, 2015]





# dependent community detection

## advantages

- a solution for the problem of instability
- improved computational complexity

## disadvantages

- traditional community detection methods are no longer directly applicable

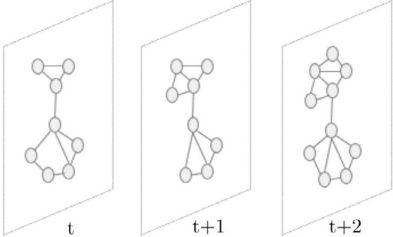
## temporal communities: idea #3

### simultaneous community detection on all snapshots

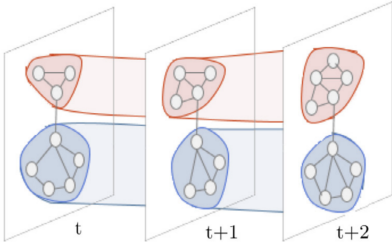
[Dakiche et al., 2019]

- construct a **static expansion graph**
  - add edges between instances of nodes in different timestamps
- run a standard community detection on the resulting graph

# simultaneous community detection on all snapshots



(1) A dynamic network consisting of three snapshots



(2) Community detection on all snapshots

[Dakiche et al., 2019]

# simultaneous community detection

## costs

- **switching cost**: each node  $u$  incurs cost  $C_{sw}$  when changing community affiliation
- **false negative cost**: two nodes incur cost  $C_{fn}$  when belong to the same community but do not interact
- **false positive cost**: two nodes incur cost  $C_{fp}$  when belong to different communities but do interact

## resulting problem

- find a partition into clusters that **minimizes** the total cost of switching, false negative, and false positive

[Tantipathananandh and Berger-Wolf, 2011]

## simultaneous community detection on all snapshots

### advantages

- provides a solution for the problem of instability

### disadvantages

- no possibility to track community evolution in a network evolving in real time

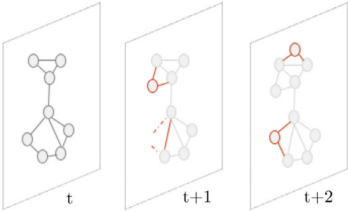
temporal communities: idea #4

dynamic community detection

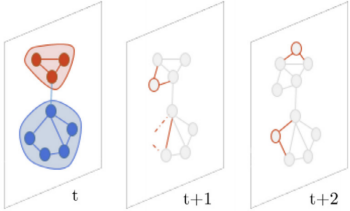
[Dakiche et al., 2019]

- update previously discovered communities according to network modifications

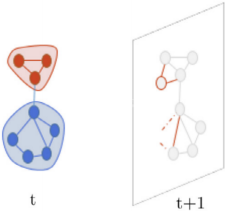
# dynamic community detection



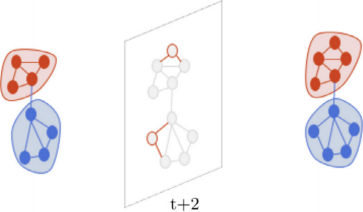
(1) Temporal network: an initial snapshot and sequence of modifications



(2) Community detection on first snapshot



(3) Update communities of  $t$  according to modifications of  $t+1$



(4) Update communities of  $t+1$  according to modifications of  $t+2$

# dynamic community detection

## advantages

- provides a solution for the problem of instability
- light-weight methods to track communities

## disadvantages

- possibility to drift towards invalid communities



event detection

## event detection

- given a network representing some kind of activity
  - network of social interactions
  - social-media feed
  - transportation network
- an event can be generally defined as an activity with some prominent **qualitative** or **quantitative difference** from the **background activity**
  - bursting news about major natural disasters
  - abnormally high traffic in the city
  - an emerging new discussion topic in social media

## temporal event detection: standard approaches

### abnormality score

- the likelihood that an interval contains an event can be estimated by comparing an abnormality score on the interval

[Heins and Stern, 2014]

### predictive models

- learn a predictive model and find intervals and time points whose behavior differ from the predicted one

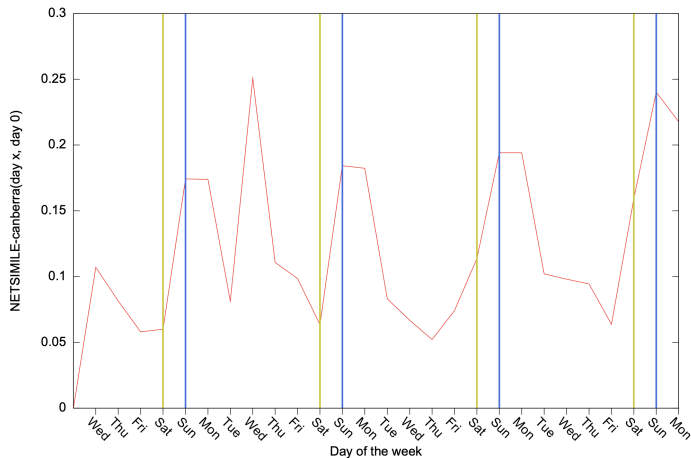
[Hunter and McIntosh, 1999, Gensler and Sick, 2017]

# Netsimile

- an event exists in  $G_{j+1}$ , if  $G_{j+1}$  is very different than  $G_j$
- for each node calculate 7 local and ego-network-based measures
  - degree
  - clustering coefficient
  - average degree of neighbours
  - average clustering coefficient of neighbours
  - number of edges in the ego-network
  - number of edges outgoing from the ego-network
  - number of neighbours of the ego-network
- combine into a signature vector and compare

[Berlingerio et al., 2012]

# Netsimile algorithm



(a) NetSimile between each day and day 0 in Yahoo! IM

[Berlingerio et al., 2012]

## spatiotemporal event detection

detailed survey by [Shi and Pun-Cheng, 2019]

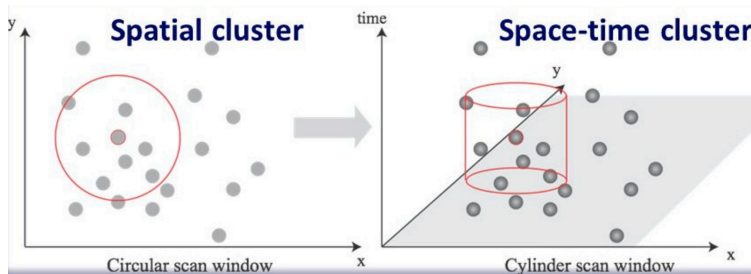
- consider time and the (geo-)location of an event
- **sources** of spatial data
  - GPS devices / smart phones
  - geo-tagged messages in online social networks
- typical approaches model the data as a set of geo-locations associated with **activity measurements**
- given a set of **locations** with activity measures, find a subset of locations that are **close** to each other and have **abnormal** activity pattern
- in **spatiotemporal** setting, one is also interested in finding the **time** interval (moment) of an event

## spatiotemporal event detection: scan statistics

- a classic family of methods is **spatial** and **spatiotemporal scan statistics**
- **scan** over the **space** and **time** windows to identify regions of data generated by some process

## spatiotemporal event detection: scan statistics

- a seminal paper : **spatial scan statistics** [Kulldorff, 1997]
- scan a circular spatial window and test the non-randomness of data against Poisson or Bernoulli baseline process
- later the approach was extended to **spatiotemporal** scans with **cylindric** windows



[Takahashi et al., 2004]



## structural event

- **structural** event:
  - set of **interconnected** abnormal **nodes**
- e.g., the edge weights represent **similarity** of nodes
  - similarities between twitter users in preferences, language, visited locations, etc.
- scan **extension** to graph model [Liu et al., 2016]
- scan through a graph **neighborhood** — a set of interconnected nodes
- **dense subgraph** detection
  - e.g., [Charikar, 2000, Khuller and Saha, 2009]

finding important nodes

# PageRank

- classic approach for measuring **node importance**
- listed in the **top-10 most important data-mining algorithms**
- numerous applications
  - ranking web pages
  - trust and distrust computation
  - finding experts in social networks
  - ...

[Wu et al., 2008]

## static PageRank

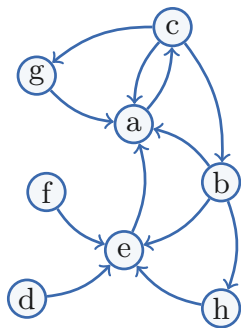
- graph  $G = (V, E)$
- corresponding row-stochastic matrix  $P \in \mathbb{R}^{n \times n}$
- personalization vector  $h \in \mathbb{R}^n$
- PageRank is the **stationary distribution** of a random walk, with restart probability  $(1 - \alpha)$

$$\pi(u) = \sum_{v \in V} \sum_{k=0}^{\infty} (1 - \alpha) \alpha^k \sum_{\substack{z \in \mathcal{Z}(v, u) \\ |z|=k}} h(v) \Pr[z | v]$$

where,  $\mathcal{Z}(v, u)$  is the set of all paths from  $v$  to  $u$

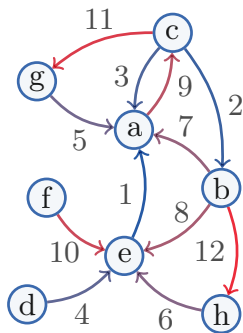
and  $\Pr[z | v] = \prod_{(i,j) \in z} P(i, j)$

# motivating example



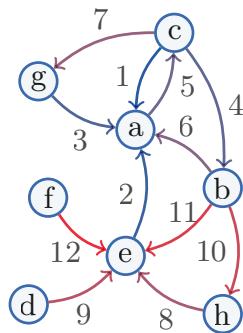
(a)

static network



(b)

temporal network

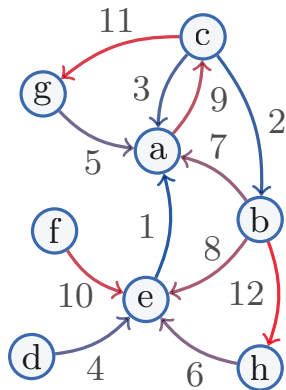


(c)

temporal network

## temporal PageRank

- make a random walk only on **temporal paths**  
e.g., **time-respecting paths**  
time-stamps increase along the path



$c \rightarrow b \rightarrow a \rightarrow c$  : time respecting

$a \rightarrow c \rightarrow b \rightarrow a$  : **not** time respecting

## temporal PageRank

- **intuition** : probability of visiting node  $u$  at time  $t$ ,  
given a random walk on temporal paths
- need to model probability of following next temporal edge
  - we use an exponential distribution
- **temporal PageRank definition**

$$r(u, t) = \sum_{v \in V} \sum_{k=0}^t (1 - \alpha) \alpha^k \sum_{\substack{z \in \mathcal{Z}^T(v, u | t) \\ |z|=k}} \Pr'[z | t]$$

$\mathcal{Z}^T(v, u | t)$  set of temporal paths from  $v$  to  $u$  until time  $t$

## static vs. temporal PageRank

- computation: simple online algorithm iterating over edges
- temporal PageRank is designed to capture changes in network dynamics and concept drifts
- proposition :
  - if the edge distribution is stable, then
  - as  $T \rightarrow \infty$ , the temporal PageRank on  $G$
  - converges to the static PageRank on  $G_S$ ,
  - with personalization vector equal to weighted out-degree

[Rozenstein and Gionis, 2016]



diffusion analysis and influence spreading

## diffusion analysis and influence spreading

- propagation models
  - used to study disease spreading or information cascade in the network
- activity spreading: virus, information, idea, rumor
- applications: epidemiology, information security, marketing
- why use models?
  - facilitate mathematical analysis of propagation processes
  - have intuitive interpretation
  - proven to be realistic by empirical studies
- extensive survey in the book [Shakarian et al., 2015]

## standard models

- susceptible-infected (SI) model
  - SIR, SIRS, other variants
- independent cascade (IC) model
- linear threshold (LT) model
- shortest path (SP) model

## susceptible-infected-recovered (SIR) model

- a popular model to analyze epidemics
- population is divided into three categories
  - **susceptible**: may be infected if comes in contact with an infectious individual
  - **infectious**: infected and capable of infecting susceptible individuals
  - **recovered**: either recovered and become immune, or deceased
- $S(t)$ ,  $I(t)$ ,  $R(t)$ : number of susceptible, infectious, recovered individuals at time  $t$
- **ordinary differential equations** describe the rate of growth of the three populations

## susceptible-infected-recovered (SIR) model

- **parameters** of the SIR model
  - $\beta$ : average number of contacts, multiplied by the probability of disease transmission
  - $\gamma = 1/D$ , where  $D$ : an individual is infectious for an average time period  $D$
  - $R_0 = \beta/\gamma$ : basic reproduction ratio
  - $\lambda$ : largest eigenvalue of stochastic system matrix, if network structure is considered
- model can be used to analyze whether the disease will persist or die out
- exhibits **threshold phenomena** behavior
- many variants of the basic model

## static models: assumptions

all models have similar implicit assumptions on temporality:

1. uniform time steps
2. interactions happen at each time step and are independent

## drawbacks of static models

- large heterogeneity in the time instances of real interactions

[Barabasi, 2005, Candia et al., 2008, Leskovec and Horvitz, 2008]

- burstiness in communication patterns
- periodic activity changes
- causal relationships between interactions

## temporal propagation models

- intuitive **extensions** from **static graphs** to temporal graphs
- add distributions (e.g., Poisson or power-law) of the **intervals between interactions**  
[Vazquez et al., 2007, Min et al., 2011]
- continuous time, partially observed graph
- develop **mathematical analysis** for novel and generalized models  
[Harris, 2002, Fernández-Gracia et al., 2011]



## typical problem formulations

- immunization strategies
- influence maximization
- seed and cascade reconstruction

## static immunization strategies

- how to stop or prevent a viral diffusion?
- **main aspects** differentiating the research works:
  - assumptions about the **spreading model**
  - assumptions about the **network structure**
  - whether the whole network is **observable**
- **both** assumptions on the network **structure** and on the infection **propagation** are **crucial**
- results may not hold for any **general network** and **real** infection

[Newman, 2003, Pastor-Satorras and Vespignani, 2002a]

## static immunization strategies

- simple **model-blind strategies**, such as **random immunization**, perform moderately **well** in different scenarios

[Pastor-Satorras and Vespignani, 2002b, Madar et al., 2004]

- better results on real-world networks can be achieved by immunizing nodes with **high connectivity**

[Pastor-Satorras and Vespignani, 2002b, Dezső and Barabási, 2002].

- requires explicit **knowledge** of the **network structure** and it is **impractical** in real applications

## static immunization strategies

- [Cohen et al., 2003] overcomes this drawback by employing **acquaintance immunization** strategy:
- immunization of **random neighbors** of **randomly** selected nodes leads to immunization of the **most central nodes** **without** knowing any global information about the network

## temporal immunization strategies

- adjust successful static strategies
- e.g., Cohen's neighborhood vaccination scheme [Lee et al., 2012]
- two vaccination strategies
- recent :
  - ask a random individual  $i$  to name its most recent contact and vaccinate this person
- weight :
  - ask a random individual  $i$  to name its most frequent contact since some time  $t$

## static influence maximization

- how to select the **initial set** of infected nodes (**seeds**), such that the **speed**, **size**, or other **spread characteristics** are **optimized**
- applications in **marketing** and **network design**
- influence maximization problem was introduced by [Kempe et al., 2003] in the **IC** and **LT** models
- find a set of  $k$  **seed nodes**, such that the **expected number** of nodes activated by the infection cascade is **maximized**

## static influence maximization

- **NP-hard** [Kempe et al., 2003]
- **simple greedy** algorithm with approximation guarantee
- **influence maximization problem** was been studied for many different variants of other models, constraints, and objective functions
- many **practical heuristics** and approximations

[Chen et al., 2009, Chen et al., 2010, Tang et al., 2014]

## temporal influence maximization

- **intuitive** approach to capture temporality:
  - **sequence** of graphs (or snapshots)
  - each **time step** of propagation corresponds to propagation over the **corresponding graph**
  - all interactions within one time step happen **simultaneously**
- related papers by [Aggarwal et al., 2012, Zhuang et al., 2013, Gayraud et al., 2015]



## temporal influence maximization

- **another** approach:
- incorporate time into the diffusion model as **distribution of intervals** between the interactions
- different types of **models** and **interval distributions**  
[Chen et al., 2012, Liu et al., 2012, Rodriguez and Schölkopf, 2012, Du et al., 2013]
- the most **realistic approachable** setting?
- an interesting research problem:
  - **infer** propagation model parameters from the data  
[Rodriguez et al., 2011, Gomez-Rodriguez et al., 2016]

## seed and cascade reconstruction

- given some **observed data** about the **infection**
  - e.g., a **small subset** of infected nodes,  
the goal is to find the **most probable seed nodes**
- other **versions**:
  - find the **most probable cascades**
- the **order** of infection (who got infected from whom)
- these works are **data-driven**:
  - it is essential that the **assumed** propagation model matches the **actual** infection flow in the network

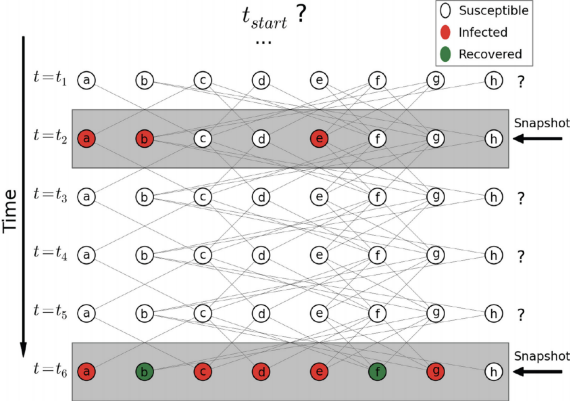
## seed and cascade reconstruction

- applications:
  - epidemiology (who was the patient zero?)
  - influencer discovery (who was the source of information?)
- a number of different approaches
  - find a single source under the SI model [Shah and Zaman, 2011]
  - multiple seeds [Prakash et al., 2012]
  - $k$  seeds under the IC model [Lappas et al., 2010]
  - take advantage of the recorded infection order [Sefer and Kingsford, 2016]
- the above papers are pre-covid, so hugely obsolete now

## temporal reconstruction

- the problems formulated **in this setting** tend to be either
  - **oversimplified** versions of static reconstruction or
  - become **too hard** or **ill-posed**
- knowing the history of interactions allow to reconstruct feasible paths of infection and prune unfeasible
- any noise or missing information adds uncertainty
  - typically need more assumptions about the **noise** and **missing information**
- the knowledge of the **diffusion model** is crucial
- see survey paper by [Holme, 2015]

# history reconstruction



[Sefer and Kingsford, 2016]

# agenda

part I : introduction and motivation

part II : models of temporal networks

part III : algorithmic frameworks

part IV : data mining problems

part V : conclusions and future challenges

part V

conclusions and future challenges

## temporal community detection: challenges

- large number of **problem formulations** and variants
- lack of **fundamental theoretical** treatment
  - most of the approaches are **heuristics**
  - many are combinations of **several** ideas and algorithms
  - require **many** parameters and attention to **implementation details**
- **hard to compare** methods and choose one for a specific application
  - **few datasets** with ground-truth temporal communities
  - synthetic generators are built on **various assumptions**
  - **no** standard benchmarks
- a large number of **quality metrics** to calculate and compare
- may be **misleading** if a method is not designed for that particular community definition








## event detection: challenges

- actively **evolving** area, **application-** and **data-oriented**
  - families of problems and methods are considered only for the **specific** sources of data
    - e.g., a large body of research is focused on the analysis of **Twitter** data
- [Atefeh and Khreich, 2015]
- **no unified classification** for problem settings, research questions, and data requirements
  - **speed and quality**:
    - **online streaming** event-detection techniques are demanded for **nearly real-time** event detection
    - **quality**: **both false events and missed events** may have a high price
  - methods should rely more **multi-modal** data, e.g., combining network structure with text

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