Introduction to Mechanism Design for Single Parameter Environments

Based on slides by V. Markakis
Mechanism Design

• What is mechanism design?
• It can be seen as reverse game theory
• **Main goal:** design the rules of a game so as to
  • avoid strategic behavior by the players
  • and more generally, enforce a certain behavior for the players or other desirable properties

• Applied to problems where a “social choice” needs to be made
  • i.e., an aggregation of individual preferences to a single joint decision

• strategic behavior = declaring false preferences in order to gain a higher utility
Examples

• Elections
  • Parliamentary elections, committee elections, council elections, etc
  • A set of voters
  • A set of candidates
  • Each voter expresses preferences according to the election rules
    • E.g., by specifying his single top choice, or by specifying his first few choices, or by submitting a full ranking of the candidates
  • Social choice: can be a single candidate (single-winner election) or a set of candidates (multi-winner election) or a ranking of the candidates
Examples

• **Auctions**
  • An auctioneer with some items for sale
  • A set of bidders express preferences (offers) over items
    • Or combinations of items
  • Preferences are submitted either through a valuation function, or according to some bidding language
  • **Social choice:** allocation of items to the bidders
Examples

• **Government policy making and referenda**
  • A municipality is considering implementing a public project
  • Q1: Should we build a new road, a library or a tennis court?
  • Q2: If we build a library where shall we build it?
  • Citizens can express their preferences in an online survey or a referendum
  • **Social choice**: the decision of the municipality on what and where to implement
Specifying preferences

- In all the examples, the players need to submit their preferences in some form.
- Representation of preferences can be done by:
  - A valuation function (specifying a value for each possible outcome)
  - A ranking (an ordering on possible outcomes)
  - An approval set (which outcomes are approved)
- Possible conflict between increased expressiveness vs complexity of decision problem.
Single-item Auctions
Auctions

Set of players
N = \{1, 2, ..., n\}
Auctions

• A means of conducting transactions since antiquity
  • First references of auctions date back to ancient Athens and Babylon

• Modern applications:
  • Art works
  • Stamps
  • Flowers (Netherlands)
  • Spectrum licences
  • Other governmental licences
  • Pollution rights
  • Google ads
  • eBay
  • Bonds
  • ...
Auctions

• Earlier, the most popular types of auctions were
  • The English auction
    • The price keeps increasing in small increments
    • Gradually bidders drop out till there is only one winner left
  • The Dutch auction
    • The price starts at $+\infty$ (i.e., at some very high price) and keeps decreasing
    • Until there exists someone willing to offer the current price
    • There exist also many variants regarding their practical implementation
• These correspond to ascending or descending price trajectories
Sealed bid auctions

• **Sealed bid**: We think of every bidder submitting his bid in an envelope, without other players seeing it
  - It does not really have to be an envelope, bids can be submitted electronically
  - The main assumption is that it is submitted in a way that other bidders cannot see it

• After collecting the bids, the auctioneer needs to decide:
  - Who wins the item?
    - *Easy! Should be the guy with the highest bid*
      • Yes in most cases, but not always
  - How much should the winner pay?
    - *Not so clear*
\[ \begin{align*} A & \rightarrow 90,1 \\ B & \rightarrow 90 \end{align*} \]
Sealed bid auctions

Why do we view auctions as games?

• We assume every player has a valuation $v_i$ for obtaining the good

• Available strategies: each bidder is asked to submit a bid $b_i$
  
  • $b_i \in [0, \infty)$
  • Infinite number of strategies

• The submitted bid $b_i$ may differ from the real value $v_i$ of bidder $i$
**First price auction**

**Auction rules**

- Let \( \mathbf{b} = (b_1, b_2, \ldots, b_n) \) the vector of all the offers

**Winner:** The bidder with the highest offer

- In case of ties: We assume the winner is the bidder with the lowest index (not important for the analysis)
- E.g. if there is a tie among bidder 2 and bidder 4, the winner is bidder 2

**Winner’s payment:** the bid declared by the winner

**Utility function of bidder** \( i \),

\[
  u_i(\mathbf{b}) = \begin{cases} 
  v_i - b_i , & \text{if } i \text{ is the winner} \\
  0 , & \text{otherwise}
  \end{cases}
\]
Incentives in the first price auction

Analysis of first price auctions

• There are *too many* Nash equilibria

• Can we predict bidding behavior?
   Is some equilibrium more likely to occur?

• Hard to tell what exactly will happen in practice but we can still make some conclusions for first price auctions

Observation: Suppose that $v_1 \geq v_2 \geq v_3 \ldots \geq v_n$. Then the profile $(v_2, v_2, v_3, \ldots, v_n)$ is a Nash equilibrium

Corollary: The first price auction provides incentives to bidders to hide their true value

• This is highly undesirable when $v_1 - v_2$ is large
Auction mechanisms

We would like to explore alternative payment rules with better properties.

**Definition:** For the single-item setting, an *auction mechanism* receives as input the bidding vector \( b = (b_1, b_2, \ldots, b_n) \) and consists of

- an **allocation algorithm** (who wins the item)
- a **payment algorithm** (how much does the winner pay)

Most mechanisms satisfy **individual rationality:**

- Non-winners do not pay anything ✓
- If the winner is bidder \( i \), her payment will not exceed \( b_i \) (it is guaranteed that no-one will pay more than what she declared)
Auction mechanisms

Aligning Incentives

• Ideally, we would like mechanisms that do not provide incentives for strategic behavior

• How do we even define this mathematically?

An attempt:

**Definition:** A mechanism is called **truthful** (or strategyproof, or incentive compatible) if for every bidder \( i \), and for every profile \( b_{-i} \) of the other bidders, it is a **dominant strategy** for \( i \) to declare her real value \( v_i \), i.e., it holds that

\[
 u_i(v_i, b_{-i}) \geq u_i(b', b_{-i}) \quad \text{for every } b' \neq v_i
\]
Auction mechanisms

• In a truthful mechanism, every rational agent knows what to play, independently of what the other bidders are doing
  • It is a win-win situation:
    • The auctioneer knows that players should not strategize
    • The bidders also know that they should not spend time on trying to find a different strategy
  • Very powerful property for a mechanism
  • Fact: The first-price mechanism is not truthful

Are there truthful mechanisms?
The 2\textsuperscript{nd} price mechanism (Vickrey auction)

[Vickrey ’61]

• Allocation algorithm: same as before, the bidder with the highest offer
  • In case of ties: we assume the winner is the bidder with the lowest index

• Payment algorithm: the winner pays the 2\textsuperscript{nd} highest bid
  • Hence, the auctioneer offers a discount to the winner

Observation: the payment does not depend on the winner’s bid!
  • The bid of each player determines if he wins or not, but not what he will pay
The 2\textsuperscript{nd} price mechanism (Vickrey auction)

[Vickrey ’61] (Nobel prize in economics, 1996)

- **Theorem:** The 2\textsuperscript{nd} price auction is a truthful mechanism ✓

Proof sketch:

- Fix a bidder i, and let \( b_{-i} \) be an arbitrary bidding profile for the rest of the players
- Let \( b^* = \max_{j \neq i} b_j \)
- Consider now all possible cases for the final utility of bidder i, if he plays \( v_i \):
  - \( v_i < b^* \) \( \Rightarrow \) i loses \( \Rightarrow u_i = 0 \)
  - \( v_i > b^* \)
  - \( v_i = b^* \)

- In all these different cases, we can prove that bidder i does not become better off by deviating to another strategy
Optimization objectives

What do we want to optimize in an auction?

Usual objectives:
- **Social welfare** (the total welfare produced for the involved entities)
- **Revenue** (the payment received by the auctioneer)

We will focus on social welfare
Optimization objectives

What do we want to optimize in an auction?

**Definition:** The utilitarian social welfare produced by a bidding vector \( \mathbf{b} \) is \( SW(\mathbf{b}) = \Sigma_i u_i(\mathbf{b}) \)

- The summation includes the auctioneer’s utility (= the auctioneer’s payment)
- The auctioneer’s payment cancels out with the winner’s payment

- For the single-item setting, \( SW(\mathbf{b}) = \text{the value of the winner for the item} \)
- An auction is *welfare maximizing* if it always produces an allocation with optimal social welfare (when bidders are truthful)
Vickrey auction: an ideal auction format

Summing up:

**Theorem:** The 2nd price auction is

- **truthful** [incentive guarantees]
- **welfare maximizing** [economic performance guarantees]
- **implementable in polynomial time** [computational performance guarantees]

Even though the valuations are private information to the bidders, the Vickrey auction solves the welfare maximization problem as if the valuations were known.
Generalizations to single-parameter environments
Single-parameter mechanisms

- In many cases, we do not have a single item to sell, but multiple items
- But still, the valuation of a bidder could be determined by a single number (e.g., value per unit)
- Note: the valuation function may depend on various other parameters, but we assume only a single parameter is private information to the bidder
  - The other parameters may be publicly known information
- We can treat all these settings in a unified manner
- Our focus: **Direct revelation mechanisms**
  - The mechanism asks each bidder to submit the parameter that completely determines her valuation function
Examples of single-parameter environments

• Single-item auctions:
  • One item for sale
  • each bidder is asked to submit **his value for acquiring the item**

• k-item unit-demand auctions
  • k identical items for sale
  • each bidder submits his value per unit and can win at most one unit

• Knapsack auctions
  • k identical items, each bidder has a value for obtaining a certain number of units

• Single-minded auctions
  • a set of (non-identical) items for sale
  • each bidder is interested in acquiring a specific subset of items (known to the mechanism)
  • Each bidder submits his value for the set she desires
Examples of single-parameter environments

- Sponsored search auctions
  - multiple advertising slots available, arranged from top to bottom
  - each bidder interested in acquiring as high a slot as possible
  - each bidder submits his value per click

- Public project mechanisms
  - deciding whether to build a public project (e.g., a park)
  - each bidder submits his value for having the project built

In all these settings, we can have multiple winners in the auction