Introduction to Mechanism Design for Single Parameter Environments

Based on slides by V. Markakis

Mechanism Design

- What is mechanism design?
- It can be seen as reverse game theory
- Main goal: design the rules of a game so as to
  - avoid strategic behavior by the players
  - and more generally, enforce a certain behavior for the players or other desirable properties
- Applied to problems where a "social choice" needs to be made
  - i.e., an aggregation of individual preferences to a single joint decision
- strategic behavior = declaring false preferences in order to gain a higher utility

Examples

- Elections
  - Parliamentary elections, committee elections, council elections, etc
  - A set of voters
  - A set of candidates
  - Each voter expresses preferences according to the election rules
    - E.g., by specifying his single top choice, or by specifying his first few choices, or by submitting a full ranking of the candidates
  - Social choice: can be a single candidate (single-winner election) or a set of candidates (multi-winner election) or a ranking of the candidates

Examples

- Auctions
  - An auctioneer with some items for sale
  - A set of bidders express preferences (offers) over items
    - Or combinations of items
  - Preferences are submitted either through a valuation function, or according to some bidding language
  - Social choice: allocation of items to the bidders

Examples

- Government policy making and referenda
  - A municipality is considering implementing a public project
  - Q1: Should we build a new road, a library or a tennis court?
  - Q2: If we build a library where shall we build it?
  - Citizens can express their preferences in an online survey or a referendum
  - Social choice: the decision of the municipality on what and where to implement

Specifying preferences

$2^{50} \approx 10^{15}$

- In all the examples, the players need to submit their preferences in some form
- Representation of preferences can be done by
  - A valuation function (specifying a value for each possible outcome)

    50 items

    3 items a, b, c

    2^3 values
  - A ranking (an ordering on possible outcomes)
  - An approval set (which outcomes are approved)
- Possible conflict between increased expressiveness vs complexity of decision problem

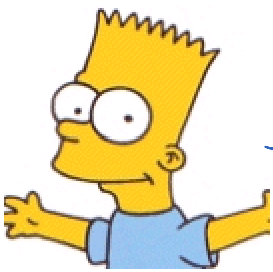
$\emptyset$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a,b\}$, $\{a,c\}$, $\{b,c\}$, $\{a,b,c\}$

0 0 0 1 10

Single-item Auctions

Auctions

b_1



b_2

b_3

Set of players
N = {1, 2, …, n}

1 indivisible good

Auctions

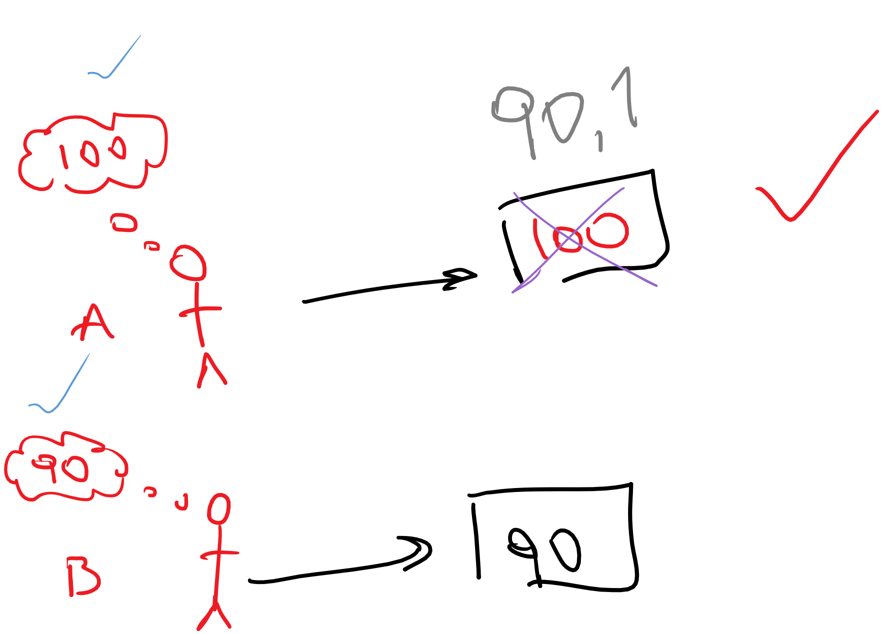
- A means of conducting transactions since antiquity
  - First references of auctions date back to ancient Athens and Babylon
- Modern applications:
  - Art works
  - Stamps
  - Flowers (Netherlands)
  - Spectrum licences
  - Other govermental licences
  - Pollution rights
  - Google ads
  - eBay
  - Bonds
  - ...

Auctions

- Earlier, the most popular types of auctions were
  - The English auction
    - The price keeps increasing in small increments
    - Gradually bidders drop out till there is only one winner left
  - The Dutch auction
    - The price starts at $+\infty$ (i.e., at some very high price) and keeps decreasing
    - Until there exists someone willing to offer the current price
  - There exist also many variants regarding their practical implementation
- These correspond to ascending or descending price trajectories

Sealed bid auctions

- Sealed bid: We think of every bidder submitting his bid in an envelope, without other players seeing it
  - It does not really have to be an envelope, bids can be submitted electronically
  - The main assumption is that it is submitted in a way that other bidders cannot see it
- After collecting the bids, the auctioneer needs to decide:
  - Who wins the item?
    - *Easy! Should be the guy with the highest bid*
      - Yes in most cases, but not always
  - How much should the winner pay?
    - *Not so clear*



90,1

$P_A = \cancel{100}$

$\simeq 10$

$u_A = \cancel{0}$

$\boxed{\text{item}}$

Sealed bid auctions

Why do we view auctions as games?

- We assume every player has a valuation v_i for obtaining the good

- Available strategies: each bidder is asked to submit a bid b_i
  - $b_i \in [0, \infty)$
  - Infinite number of strategies

- The submitted bid b_i may differ from the real value v_i of bidder i

First price auction

Auction rules

- Let $\mathbf{b} = (b_1, b_2,\ldots, b_n)$ the vector of all the offers
- Winner: The bidder with the highest offer
  - In case of ties: We assume the winner is the bidder with the lowest index (not important for the analysis)
  - E.g. if there is a tie among bidder 2 and bidder 4, the winner is bidder 2
- Winner's payment: the bid declared by the winner
- Utility function of bidder i,

why say the truth??

$$u_i(\mathbf{b}) = \begin{cases} v_i - b_i\,, & \text{if i is the winner} \\ 0, & \text{otherwise} \end{cases}$$

Incentives in the first price auction

Analysis of first price auctions

- There are *too many* Nash equilibria
- Can we predict bidding behavior?
  Is some equilibrium more likely to occur?

- Hard to tell what exactly will happen in practice but we can still make some conclusions for first price auctions

Observation: Suppose that $v_1 \geq v_2 \geq v_3 \ldots \geq v_n$. Then the profile $(v_2, v_2, v_3, \ldots, v_n)$ is a Nash equilibrium

Corollary: The first price auction provides incentives to bidders to hide their true value

- This is highly undesirable when $v_1 - v_2$ is large

Auction mechanisms

We would like to explore alternative payment rules with better properties

Definition: For the single-item setting, an auction mechanism receives as input the bidding vector **b** = $(b_1, b_2, \dots, b_n)$ and consists of

- an allocation algorithm (who wins the item)
- a payment algorithm (how much does the winner pay)

Most mechanisms satisfy individual rationality:

no negative utilities (under truthful bidding)

- Non-winners do not pay anything
- If the winner is bidder i, her payment will not exceed b_i (it is guaranteed that no-one will pay more than what she declared)

Auction mechanisms

Aligning Incentives

- Ideally, we would like mechanisms that do not provide incentives for strategic behavior
- How do we even define this mathematically?

Telling the truth is a PNE -

An attempt:

Definition: A mechanism is called truthful (or strategyproof, or incentive compatible) if for every bidder i, and for every profile $\mathbf{b}_{-i}$ of the other bidders, it is a **dominant strategy** for i to declare her real value v_i, i.e., it holds that

$$u_i(v_i, \mathbf{b}_{-i}) \geq u_i(b', \mathbf{b}_{-i}) \text{ for every } b' \neq v_i$$

Auction mechanisms

- In a truthful mechanism, every rational agent knows what to play, independently of what the other bidders are doing
- It is a win-win situation:
  - The auctioneer knows that players should not strategize
  - The bidders also know that they should not spend time on trying to find a different strategy
- Very powerful property for a mechanism
- Fact: The first-price mechanism is not truthful

Are there truthful mechanisms?

The 2nd price mechanism (Vickrey auction)

[Vickrey '61]

- **Allocation algorithm:** same as before, the bidder with the highest offer
  - In case of ties: we assume the winner is the bidder with the lowest index
- **Payment algorithm:** the winner pays the *2nd highest bid*
- Hence, the auctioneer offers a discount to the winner

Observation: the payment does not depend on the winner's bid!

- The bid of each player determines if he wins or not, but not what he will pay

The 2nd price mechanism (Vickrey auction)

[Vickrey '61] (Nobel prize in economics, 1996)

- **Theorem:** The 2nd price auction is a truthful mechanism ✓

Proof sketch:

- Fix a bidder i, and let $\mathbf{b}_{-i}$ be an arbitrary bidding profile for the rest of the players
- Let $b^* = \max_{j \neq i} b_j$ b_i b^*: highest bid, not i
- Consider now all possible cases for the final utility of bidder i, if he plays v_i
  - $v_i < b^*$ $\Rightarrow$ i losses $\Rightarrow u_i = 0$
    - bid $b_i > v_i$? $\rightarrow$ $b_i > b^* \Rightarrow u_i = v_i - b^* < 0$; $b_i < b^*$
    - bid $b_i < v_i$? $\Rightarrow u_i = 0$
  - $v_i > b^*$ ✓
  - $v_i = b^*$ ✓
  - In all these different cases, we can prove that bidder i does not become better off by deviating to another strategy

Optimization objectives

What do we want to optimize in an auction?

sum of values of the winners

Usual objectives:

sum of utilities of everyone or

- Social welfare (the total welfare produced for the involved entities)
- Revenue (the payment received by the auctioneer)

We will focus on social welfare

Optimization objectives

What do we want to optimize in an auction?

Definition: The utilitarian social welfare produced by a bidding vector **b** is SW(**b**) = $\Sigma_i\, u_i(\mathbf{b})$

- The summation includes the auctioneer's utility (= the auctioneer's payment)
- The auctioneer's payment cancels out with the winner's payment

➢ For the single-item setting, SW(**b**) = the value of the winner for the item

➢ An auction is welfare maximizing if it always produces an allocation with optimal social welfare (when bidders are truthful)

Vickrey auction: an ideal auction format

Summing up:

Theorem: The 2^{nd} price auction is

- truthful [incentive guarantees]
- welfare maximizing [economic performance guarantees]
- implementable in polynomial time [computational performance guarantees]

Even though the valuations are private information to the bidders, the Vickrey auction solves the welfare maximization problem as if the valuations were known

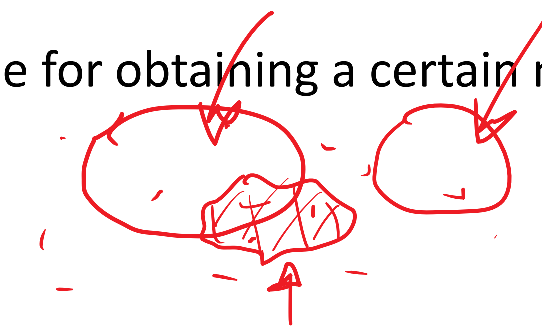
Generalizations to single-parameter environments

Single-parameter mechanisms

- In many cases, we do not have a single item to sell, but multiple items
- But still, the valuation of a bidder could be determined by a single number (e.g., value per unit)
- Note: the valuation function may depend on various other parameters, but we assume only a single parameter is private information to the bidder
  - The other parameters may be publicly known information
- We can treat all these settings in a unified manner
- Our focus: **Direct revelation** mechanisms
  - The mechanism asks each bidder to submit the parameter that completely determines her valuation function

Examples of single-parameter environments

- **Single-item auctions:**
  - One item for sale
  - each bidder is asked to submit his value for acquiring the item
- **k-item unit-demand auctions**
  - k identical items for sale
  - 10
  - each bidder submits his value per unit and can win at most one unit
- **Knapsack auctions**
  - k identical items, each bidder has a value for obtaining a certain number of units



- **Single-minded auctions**
  - a set of (non-identical) items for sale
  - each bidder is interested in acquiring a specific subset of items (known to the mechanism)
  - Each bidder submits his value for the set she desires

Examples of single-parameter environments

Myerson's Lemma

- Sponsored search auctions
  - multiple advertising slots available, arranged from top to bottom
  - each bidder interested in acquiring as high a slot as possible
  - each bidder submits his value per click
- Public project mechanisms
  - deciding whether to build a public project (e.g., a park)
  - each bidder submits his value for having the project built

In all these settings, we can have multiple winners in the auction