Social Networks and Online Markets

Homework 3

Due: 14/6/2020, 23:59

Instructions

You must hand in the homeworks electronically and before the due date and time.

This homework has to be done by each **person individually**.

Handing in: You must hand in the homeworks by the due date and time by an email to `aris@diag.uniroma1.it` and `amanatidis@diag.uniroma1.it` that will contain as attachment **(not links to some file-uploading server!)** a .pdf file with your answers.
After you submit, you will receive an acknowledgment email that your homework has been received and at what date and time. If you have not received an acknowledgment email within 2 days after you submit then contact Aris.

The solutions for the theoretical exercises must contain your answers either typed up or hand written clearly and scanned.

For information about collaboration, and about being late check the web page.

Problem 1. Each of you will submit a (rational) number between 0 and 100. We will then compute the *average* (arithmetic mean) of all the numbers submitted and multiply that number with 2/3. Whoever got closest to this latter number wins the game and earns 10% bonus points on this assignment.

1. Are there any pure Nash equilibria in this game? Justify your answer.
2. Submit your bid. Is your bid "predicted" by an equilibrium? If not, briefly explain (in 2-3 lines) why you bid like this.

Problem 2. Here we focus on the expected revenue of the seller (auctioneer) in a Vickrey auction (also known as second-price auction).

Assume that the seller knows that there are n *rational* bidders who have independent, private values which are either 1 or 3. In particular, for each bidder i, $v_i = 1$ with probability 1/2 and $v_i = 3$ with probability 1/2. (Ties are always resolved according to some fixed deterministic tie-breaking rule.) We use R to denote the seller's revenue in a single run of the Vickrey auction. Assume that the item has no value for the seller.

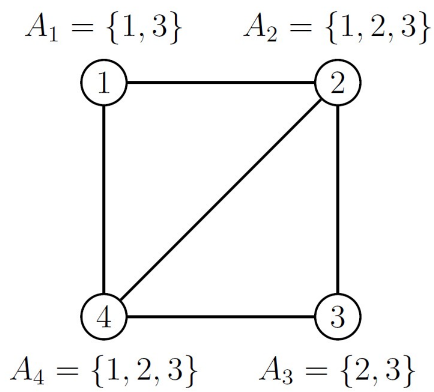
1. Show that for $n = 2$ the seller's expected revenue $\mathbb{E}(R)$ is equal to 3/2. What is the seller's expected revenue for $n = 3$?
2. Calculate $\lim_{n\to\infty} \mathbb{E}(R)$.

Problem 3. In a *coordination game* we are given an undirected (non-empty) graph $G = (N, E)$ with a set $N = \{1, \dots, n\}$ of $n \geq 2$ nodes and a set $X = \{1, \dots, k\}$ of $k \geq 1$ colors. Each node $i \in N$ corresponds to a player who chooses a color from a non-empty set of colors $A_i \subseteq X$ available to her. Note that the color sets of the players are *not* necessarily identical. The goal of each player $i \in N$ is to choose a color $a_i \in A_i$ such that the number of edges to neighbors having the *same* color is maximized. More formally, given an action profile $\mathbf{a} = (a_1, \ldots, a_n) \in A_1 \times \cdots \times A_n$, the utility $u_i(\mathbf{a})$ of player i is defined as

$$u_i(\mathbf{a}) = |\{\{i,j\} \in E \ : \ a_i = a_j\}|.$$

Define the *social welfare* of a strategy profile $\mathbf{a}$ as $SW(\mathbf{a}) = \sum_{i \in N} u_i(\mathbf{a})$. Let $\mathbf{a}^*$ denote a social optimum of maximum social welfare. The price of anarchy is said to be at least $\alpha \in [0,1]$ if $SW(\mathbf{a}) \geq \alpha SW(\mathbf{a}^*)$ for every pure Nash equilibrium $\mathbf{a}$.

1. Consider the instance of a coordination game depicted below with $n = 4$ players and $k = 3$ colors. Determine the price of anarchy of this instance. Justify your answer.



2. Show that in general the price of anarchy of coordination games is no better than 0. Prove that this claim holds even if the graph G is given arbitrarily.