Social Networks and Online Markets

Homework 1

Due: 25/6/2019, 23:59

Instructions

You must hand in the homeworks electronically and before the due date and time.

The first homework has to be done by each **person individually**.

Handing in: You must hand in the homeworks by the due date and time by an email to `aris@dis.uniroma1.it` that will contain as attachment **(not links to some file-uploading server!)** a .zip or .pdf file with your answers.
After you submit, you will receive an acknowledgement email that your homework has been received and at what date and time. If you have not received an acknowledgement email within 2 days after you submit then contact Aris.

The solutions for the theoretical exercises must contain your answers either typed up or hand written clearly and scanned.

For information about collaboration, and about being late check the web page.

Problem 1. We create a small-world graph G according to the following model. We start with a *directed* cycle with n nodes. That is, we have nodes $v_1, v_2, \ldots v_n$ and we have a directed edge from each v_i to v_{i+1} (v_n is connected to v_1). Each of these edges has length 1. In addition there exists another "central" node v, which is connected to each of the nodes v_i with probability p, each choice being mutually independent from all the other choices. Each edge (v, v_i) that exists is undirected and has length 1/2. In other words, we create some shortcuts of length 1 between some pairs of nodes (those for which an edge exists).

1. Consider two nodes v_i, v_j on the cycle, such that the distance from v_i to v_j on the cycle is k. Let $P(\ell, k)$ be the probability that the shortest path between v_i and v_j is exactly ℓ. Calculate $P(\ell, k)$.
2. Compute the distribution $P(\ell)$ of the shortest path between the nodes on the cycle.
3. **Optional, bonus:** Compute the average distance between the nodes on the graph. Assume that n is sufficiently large.

Problem 2. Consider the following modification of the Barabassi–Albert preferential attachement model that we did in class: When a new node arrives at time t again it comes with ℓ edges. However, this time each edge selects a node v with probability proportional to the degree d_v plus a constant c, that is, the probability equals

$$\frac{d_v + c}{(t-1)(2\ell + c)},$$

where $c \geq -\ell$, as we describe at the end of Chapter 4 in the notes (so for $c = 0$ this is the Barabassi-Albert model). Show that the degree distribution that we obtain as $t \to \infty$ is approximately a power law with exponent $3 + c/\ell$.

Problem 3. Consider the following auction. We have a seller who puts in sale an item with value \$100 (say a \$100 bill). The players make ascending bids and the rule is that who eventually makes the highest bid will take the item, however the seller gets the first two bids. Assume that bids must be made with increments of multiples of \$1.

1. Assume that we have 2 players, and each player increases (or not) his bid performing best response. How much will the seller obtain?

2. Find all the Nash equilibria of this game.

Problem 4. You are given a bipartite graph $G(L,R,E)$. The nodes of L arrive online, i.e. when $v \in L$ appears, all edges incident to v are revealed.

The ranking algorithm consists of selecting a uniform random permutation Π of the nodes of R. When v arrives, it is matched to first free neighbor according to the ordering of Π, if any exists, otherwise it is unmatched. This algorithm has a competitive ratio of $\frac{e}{e-1} \approx 1.582$ on expectation.

Now, let us assume that the nodes of L arrive in random order, i.e. the next node of L is selected uniformly at random.

- Show that ranking (for uniform random arrivals) is never better than $6/5 = 1.2$ on expectation.

  Hint: Consider the 2-regular graph on 6 nodes.

- Now study the ranking algorithm on the line graph (i.e. an $n-1$ length path). (Alternatively, consider a cycle of arbitrary length). Show that ranking performs better than $\frac{e}{e-1}$.

Problem 5.

You are given a set of sellers S and buyers B, where $|S| = |B|$. Every buyer has an evaluation for the items of S, i.e. $p(i) = \{8,9,3\}$ means that buyer i values item 1 as 8, item 2 as 9, and item 3 as 3. The valuation matrix is given below:

Sellers\Buyers	1	2	3	4	5	6
1	10	12	8	6	7	10
2	12	8	9	13	11	13
3	11	12	5	8	7	9
4	13	10	4	10	11	12
5	10	14	4	9	5	10
8	15	12	9	12	11	14

- Compute a set of market clearing prices. In particular, implement your algorithm (in a programming language of your choice) and submit as your solution the code (zip file), the final prices, and the final matching.

Problem 6. Solve book questions 15.10.1, 15.10.2, 15.10.3, 15.10.5.