Social Networks and Online Markets

Homework 1

Due: 6/5/2018, 23:59

Instructions

You must hand in the homeworks electronically and before the due date and time.

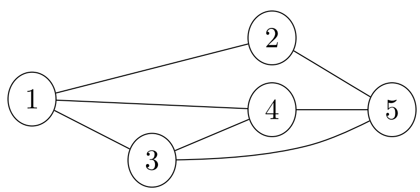
The first homework has to be done by each **person individually**.

Handing in: You must hand in the homeworks by the due date and time by an email to `aris@dis.uniroma1.it` that will contain as attachment **(not links to some file-uploading server!)** a .zip or .pdf file with your answers.
After you submit, you will receive an acknowledgement email that your homework has been received and at what date and time. If you have not received an acknowledgement email within 2 days after you submit then contact Aris.

The solutions for the theoretical exercises must contain your answers either typed up or hand written clearly and scanned.

For information about collaboration, and about being late check the web page.

Problem 1. Consider the following graph:



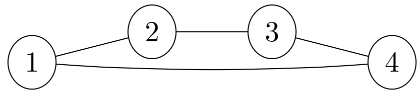
1. Compute the closeness centrality of each node.
2. Compute the node betweenness centrality of nodes 2 and 3.
3. Compute the link betweenness centrality of the links (3,4) and (2,5).

Problem 2.

Let $\mathbf{A}$ be the adjacency matrix of an undirected graph on n vertices and let $\mathbf{1}$ be the column vector whose n elements are all 1. In terms of the quantities $\mathbf{A}$ and $\mathbf{1}$, write algebraic expressions for:

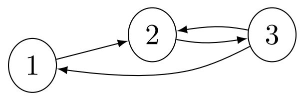
1. the vector $\mathbf{k}$ whose elements are the degrees k_i of the vertices;
2. the number m of edges in the graph;
3. the matrix $\mathbf{N}$ whose element N_{ij} is equal to the number of common neighbors of vertices i and j;
4. the total number T of triangles in the network, where a triangle is a clique of size 3, that is, three vertices, each adjacent to both of the others.

Problem 3. Consider the following graph:



1. Compute all the eigenvalues and eigenvectors of its adjacency matrix.
2. Compute all the eigenvalues of its Laplacian matrix.

Problem 4. Consider the following directed graph:



1. Compute (exactly or approximately) the Katz centrality vector with $\alpha = 0.5, \beta = 1$, with initial vector $x(0) = [1, 1, 1]$.
2. Compute (exactly or approximately) the PageRank vector with $\alpha = 0.5, \beta = 1$, with initial vector $x(0) = [1, 1, 1]$.
3. Let A be the adjacency matrix of this digraph. Explain whether the power method applied to the matrix $A^\top$ is guaranteed to converge or not to the eigenvector centrality on this digraph when $x(0) = [1, 1, 1]$.