# Social Networks and Online Markets

## Homework 1

**Due:** 22/4/2018, 23:59

**Instructions**

You must hand in the homeworks electronically and before the due date and time.

The first homework has to be done by each **person individually**.

**Handing in:** You must hand in the homeworks by the due date and time by an email to `aris@dis.uniroma1.it` that will contain as attachment **(not links to some file-uploading server!)** a .zip or .pdf file with your answers.
After you submit, you will receive an acknowledgement email that your homework has been received and at what date and time. If you have not received an acknowledgement email within 2 days after you submit then contact Aris.

The solutions for the theoretical exercises must contain your answers either typed up or hand written clearly and scanned.

For information about collaboration, and about being late check the web page.

**Problem 1.** Solve the book exercises: 3.7.4, 4.6.2, 4.6.4, and 5.6.3(c).

**Problem 2.** Show that for the Watts-Strogatz small-world model for $p = 0$ and as $k,n \to \infty$ the clustering coefficient approaches 3/4. Recall that $n$ is the number of nodes, $k$ is the degree, and $p$ the rewiring probability.

**Problem 3.** We create a small-world graph $G$ according to the following model. We start with a *directed* circle with $n$ nodes. That is, we have nodes $v_1, v_2, \ldots v_n$ and we have a directed edge from each $v_i$ to $v_{i+1}$ ($v_n$ is connected to $v_1$). Each of these edges has length 1. In addition there exists another "central" node $v$, which is connected to each of the nodes $v_i$ with probability $p$, each choice being mutually independent from all the other choices. Each edge $(v, v_i)$ that exists is undirected and has length 1/2. In other words, we create some shortcuts of length 1 between some pairs of nodes (those for which an edge exists).

- Consider two nodes $v_i, v_j$ on the circle, such that the distance from $v_i$ to $v_j$ on the circle is $k$. Let $P(\ell, k)$ be the probability that the shortest path between $v_i$ and $v_j$ is exactly $\ell$. Calculate $P(\ell, k)$.
- Compute the distribution $P(\ell)$ of the shortest path between the nodes on the circle.
- **Optional, bonus:** Compute the average distance between the nodes on the graph. Assume that $n$ is sufficiently large.