

Course : Data mining
Topic : Rank aggregation

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reading

Cynthia Dwork, Ravi Kumar, Moni Naor, D. Sivakumar:
Rank aggregation methods for the web.
WWW 2001

(optional)

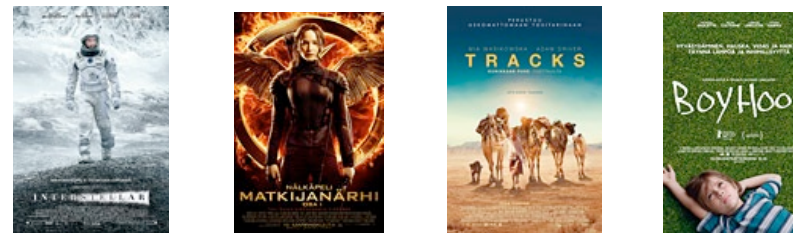
Nir Ailon, Moses Charikar, Alanthan Newman:
Aggregating inconsistent information: Ranking and clustering.
JACM 55(5), 2008

rank aggregation and voting

how can multiple agents **aggregate** their **preferences** and make a **consensus** decision?

example : three friends want to go to the cinema

Luca :



Stefano :



Aris :



which movie should they choose?

what are good properties for a voting system?



question considered by marquis de Condorcet (1743-1794)

French philosopher, mathematician and political scientist

proposed a criterion that voting systems should satisfy

known as the Condorcet criterion

what are good properties for a voting system

the Condorcet criterion

if item i defeats every other item in a pairwise majority vote, then i should be ranked first

extended Condorcet criterion

if all items in a set X defeat in pairwise comparisons all items in the set Y then the items in X should be ranked above those in Y

not all voting systems satisfy the Condorcet criterion!

the Borda count voting system



proposed by Jean-Charles de Borda
(1733-1799)

French mathematician, physicist, political
scientist, and sailor

very popular and widely-used system

the Borda count voting system

in each preference list, assign to item i **number of points** equal to the number of item it defeats

first position gets $n-1$ points, second $n-2$, ..., last 0 points

the **total weight** of i is the number of points it accumulates from all preference lists

order items in **decreasing** weight

Borda count satisfies a number of desirable properties, but **not** the Condorcet criterion

more recent attempts to design axiomatic voting systems



objective :

construct a **voting system** that satisfies a
set of **natural axioms**

Kenneth Arrow, PhD thesis, 1963

Nobel prize in economics, 1972, for
general economics equilibrium theory
and welfare theory

Arrow's axioms

non-dictatorship : the preferences of an individual should not become the group ranking without considering the preferences of others

unanimity (or **Pareto optimality**) : if every individual prefers one choice to another, then the group ranking should do the same

freedom from irrelevant alternatives : if a choice is removed, then the others' order should not change

impossibility of voting



Arrow's theorem :

it is **impossible** to construct a voting system that satisfies the previous set of three axioms

impossibility of voting

Arrow's axioms

freedom from irrelevant alternatives : if a choice is removed, then the others' order should not change

heavily disputed axiom

Borda count violates this axiom

still..

despite theoretical impossibility, the problem appears in **practice** and needs to be addressed

selecting representatives in **elections**

meta-search engines

meta-search engines

aggregate rankings from different search engines

obtain better results than any individual one

robust to spam

Google

santa


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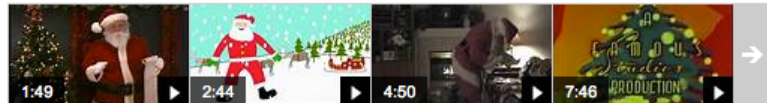
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the rank-aggregation problem

input

n items (movies, candidates, urls)

k preference lists (orderings) on the items

goal

find a single preference list that respects / agrees as much as possible with the input preference lists

Kemeny optimal aggregation



John Kemeny (1926-1992)

Hungarian-American mathematician and
computer scientist

provided a specific **formulation** of the
rank-aggregation problem

(also invented BASIC)

Kemeny optimal aggregation

input

n items (movies, candidates, urls)

k preference lists (orderings) on the items

goal

find a single preference list that minimizes the total number of out-of-order pairs

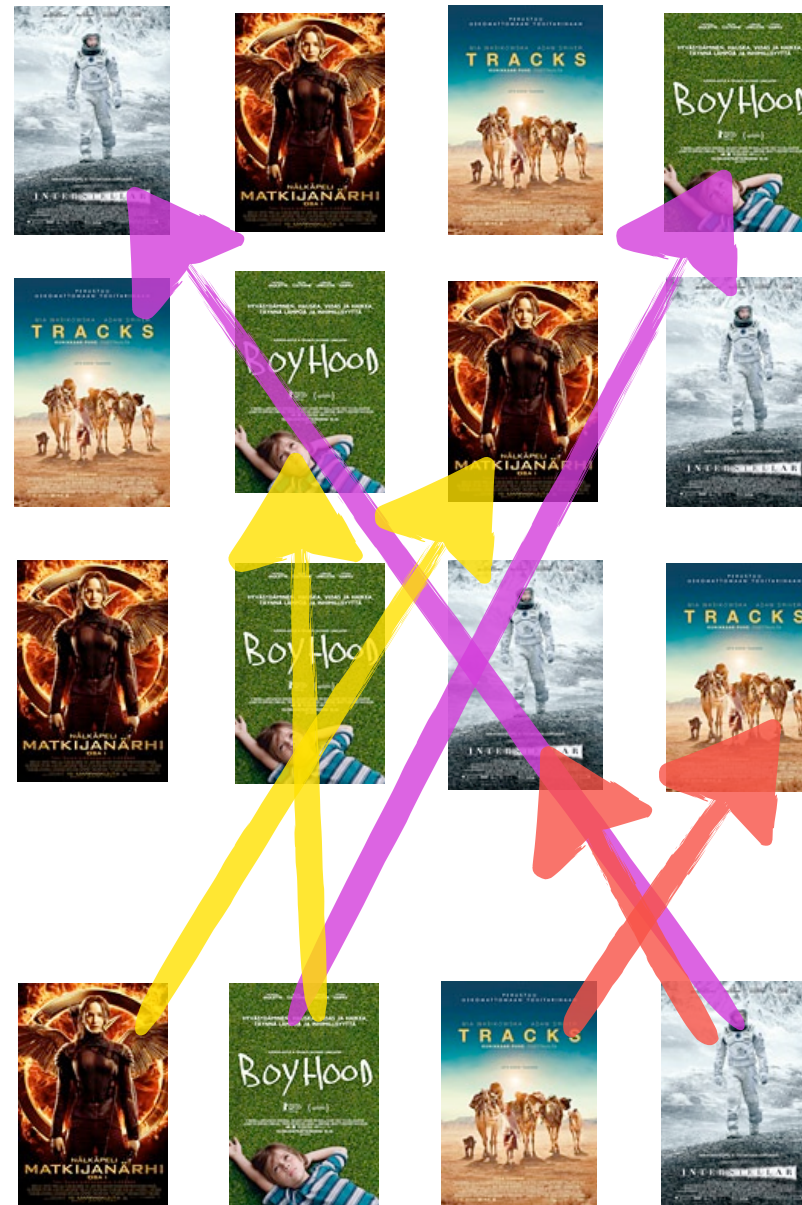
Kemeny optimal aggregation

Luca :

Stefano :

Aris :

aggregation :



preference lists

set of items U

assume n items

a **preference list** is a **bijection** (1-to-1 function) from U to $\{1, \dots, n\}$

for a preference list σ and item i in U denote by $\sigma(i)$ the **rank** (**order**) of i in σ

preference lists can be:

full, **partial**, **top-d**

distances between preference lists

consider preference lists σ and τ over the same set of items U

how similar are σ and τ ?

define a distance function

Spearman footrule distance

given two lists σ and τ over U , the Spearman footrule distance is defined as

$$F(\sigma, \tau) = \sum_{i \in U} |\sigma(i) - \tau(i)|$$

Spearman footrule distance example



$$F(\text{Luca}, \text{Stefano}) = 8$$

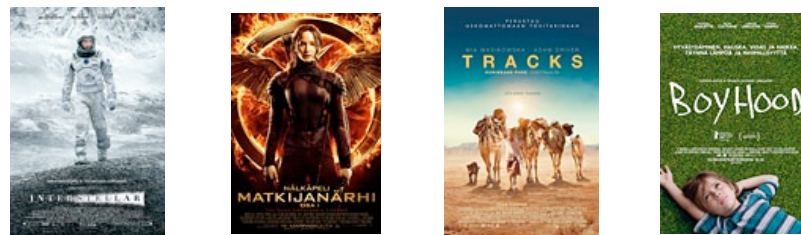
Kendall-tau distance

given two lists σ and τ over U , the Kendall-tau distance is the number of pair-wise disagreements

$$K(\sigma, \tau) = |\{(i, j) \text{ such that } \sigma(i) < \sigma(j) \text{ but } \tau(i) > \tau(j)\}|$$

Kendall-tau distance example

Luca :



Stefano :



$$K(\text{Luca}, \text{Stefano}) = 5$$

properties of Spearman footrule and Kendall-tau distances

are they **metric**?

definitions for **full** preference lists

what about **partial** lists?

the two distances **F** and **K** are **related**

for any two full preference lists:

$$K(\sigma, \tau) \leq F(\sigma, \tau) \leq 2K(\sigma, \tau)$$

the rank-aggregation problem

input

set U of n items

k preference lists τ_1, \dots, τ_k

a distance function D between preference lists

(e.g., F or K)

goal

find preference list τ_0 that minimizes total disagreement

$$D(\tau_0, \tau_1 \dots \tau_k) = \sum_{i=1 \dots k} D(\tau_0, \tau_i)$$

when $D=K$, this is Kemeny optimal aggregation

rank-aggregation with Spearman footrule distance

when distance is **F** the rank aggregation problem can be solved
in **polynomial time**

Luca :



Stefano :



Aris :



rank-aggregation with Kendall-tau distance

when distance is K and $k \geq 4$ the rank aggregation problem is **NP-hard**!

but optimal preference list with Spearman footrule distance gives **factor 2 approximation**

τ_F : optimal list according to Spearman footrule

τ_0 : optimal list according to Kendall-tau

$$K(\tau_F, \tau_1 \dots \tau_k) \leq F(\tau_F, \tau_1 \dots \tau_k) \leq F(\tau_0, \tau_1 \dots \tau_k) \leq 2K(\tau_0, \tau_1 \dots \tau_k)$$

rank-aggregation with Kendall-tau distance

any other way to get a factor-2 approximation?

l -median problem in a metric space

algorithm : pick-the-best

try each one of T_1, \dots, T_k as a potential solution and pick the best

algorithm pick-the-best is a factor 2 approximation

assume optimal solution τ_0

assume algorithm picked τ_j

assume τ_x is closest to τ_0 among all τ_1, \dots, τ_k

$$\begin{aligned} D(\tau_j, \tau_1 \dots \tau_k) &\leq D(\tau_x, \tau_1 \dots \tau_k) \\ &= \sum_{i=1 \dots k} D(\tau_x, \tau_i) \\ &\leq \sum_{i=1 \dots k} (D(\tau_x, \tau_0) + D(\tau_0, \tau_i)) \\ &= \sum_{i=1 \dots k} D(\tau_x, \tau_0) + \sum_{i=1 \dots k} D(\tau_0, \tau_i) \\ &\leq \sum_{i=1 \dots k} D(\tau_0, \tau_i) + \sum_{i=1 \dots k} D(\tau_0, \tau_i) = 2 D(\tau_0, \tau_1 \dots \tau_k) \end{aligned}$$

yet another algorithm

KwikSort [Ailon et al]

inspired by QuickSort

view data as a tournament over items in U

tournament: complete directed graph

for each pair i and j in U ,

if the majority of preference lists prefer i over j put a directed edge from i to j

the KwikSort algorithm

pick a random element i in U

put at the left L all items that point to i

put at the right R all items that i points to

recurse on L and R

KwikSort gives a factor 3 approximation

but...

...taking the best of pick-the-best and KwikSort gives a factor $6/5$ approximation!

Kemeny optimality and Condorcet criterion

Kemeny optimal aggregation satisfies the Condorcet criterion
but it is NP-hard to compute

can we have any other aggregation system that satisfies the
Condorcet criterion?

locally Kemeny optimal aggregation

a ranking τ is **locally Kemeny optimal** if there is **no bubble-sort swap** of two **consecutively placed items** that produces a ranking τ' such that

$$K(\tau', \tau_1 \dots \tau_k) \leq K(\tau, \tau_1 \dots \tau_k)$$

locally Kemeny optimal is **not necessarily** Kemeny optimal

locally Kemeny optimal aggregation

locally Kemeny optimal aggregation can be computed in polynomial time

proceed **iteratively**: in each iteration insert item **i** in the bottom of the list

bubble it up until there is item **j** such that the majority places **j** over **i**

locally Kemeny optimal aggregation satisfies the Condorcet and extended Condorcet criterion

can be applied as **post-processing** to any rank aggregation system